Material Failure Criteria

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Material failure

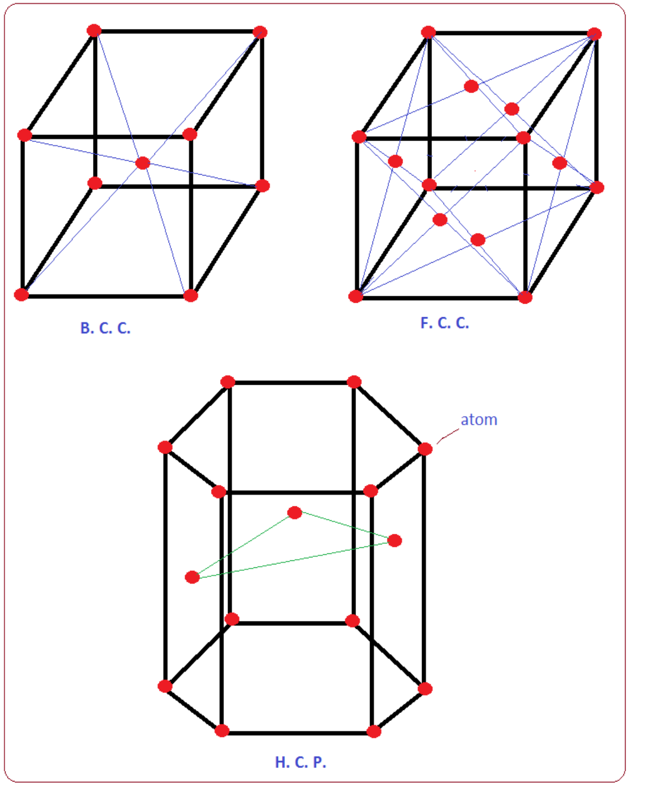
Metals solidify into crystalline structures. Their smallest unit of the lattice is the characteristic cell – building block like a brick. These building blocks can be: FCC, BCC, HCP

FCC: Face-Centered Cubic

BCC: Body-Centered Cubic

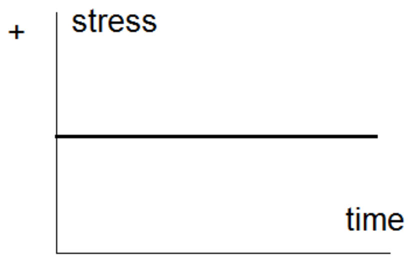
HCP: Hexagonal Close-Packed

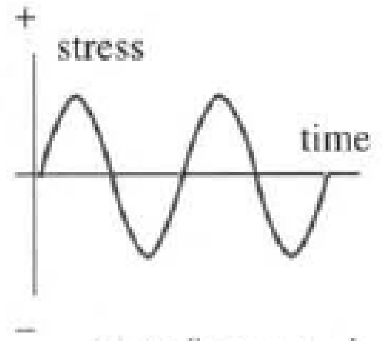
These crystal structures affect properties.



Failure in metal structures

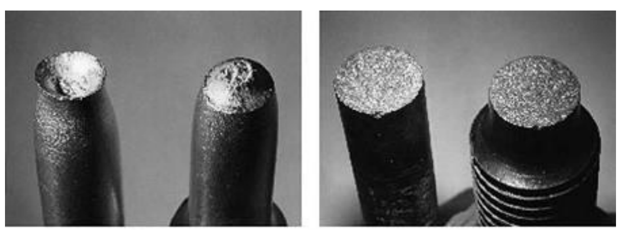
1. Static loading vs. fatigue loading





2. Brittle vs. Ductile failure mechanism

Ductile



Brittle

a) Unidirectional static loading

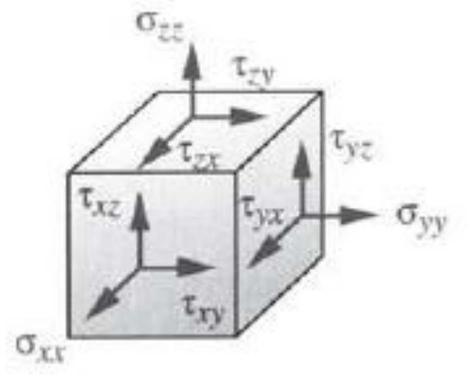
Max stress theory:

In this case: $\sigma = \frac{P}{A}$

and failure criterion is: $\sigma > S_y$

b) Triaxial stress state static loading

In general, nine stress components act on an element and we use following criteria:



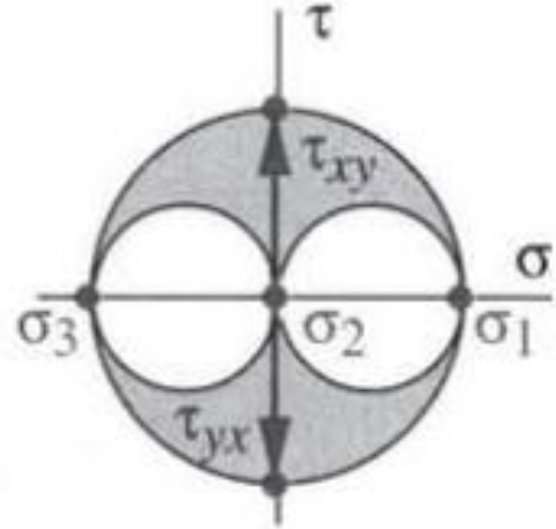
1. Max shear stress theory (Tresca)

2. Distortion energy theory (Von Mises) – It is related to deviatoric term of stress in material not hydrostatic term

Max shear stress theory (Tresca)

Material yields in shear.

Using Mohr's circle for torsion:



$$\tau_{\max} = \frac{\sigma_{max} - \sigma_{min}}{2}$$

shear yield stress: $S_{s_y} = \frac{S_y}{2}$

Failure criterion: $\tau_{\max} > S_{s_y}$

Distortion Energy Theory (Von Mises)

In this theory "Von Mises effective stress" is defined to represent the stress combination:

$$2(\sigma')^2 = (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2$$

$$\rightarrow \sigma' = \sqrt{\frac{1}{2}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]}$$

In plane stress state, σ' is reduced to:

$$\sigma' = \sqrt{\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2}$$

Failure criterion: $\sigma' > S_y$

مثال

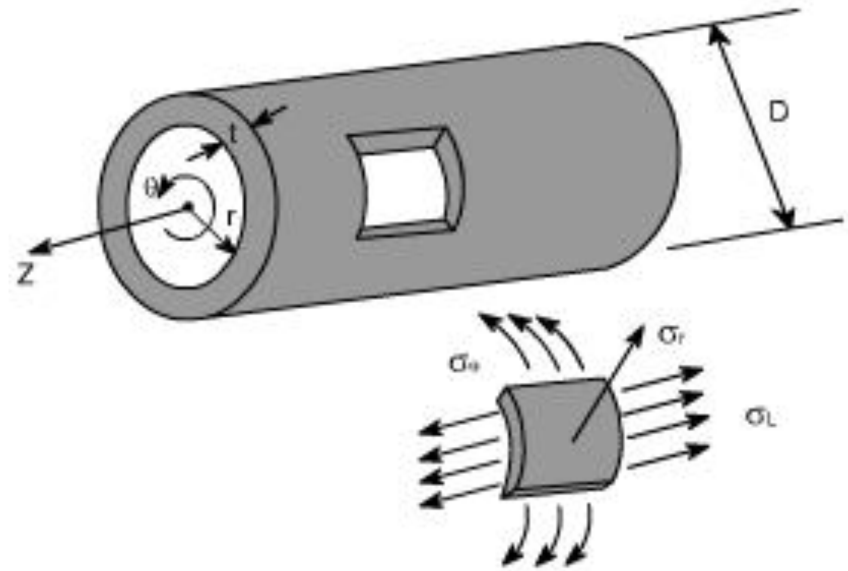
برای یک سیلندر جدارنازک با قطر 20 اینچ و ضخامت دیواره ۰.۱ اینچ، ماکزیمم فشار مخزن قبل از تسلیم چقدر می تواند باشد؟ ماده سیلندر دارای خواص elastic-perfectly plastic با استحکام تسلیم psi ۲۰۰۰۰ می باشد.

در مسئله سیلندر جدارنازک با مسئله تنش صفحه ای رو به رو هستیم و داریم:

$$\sigma_r = \sigma_{min} = 0$$

$$\sigma_h = \sigma_{max} = \frac{pr}{t}$$

$$\sigma_l = \frac{pr}{2t}$$



Tresca criterion:

$$\tau_{\max} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{\sigma_h - 0}{2}$$

$$S_{s_y} = \frac{S_y}{2} = \frac{20000}{2} = 10000\,psi$$

$$\Rightarrow \tau_{\max} = S_{s_y} = 10000 = \frac{\sigma_h}{2}$$

$$\sigma_h = \frac{pr}{t} = \frac{p \times 10}{0.1} = 20000\,psi \Rightarrow p = 200\,psi$$

Von Mises criterion:

$$\sigma' = \sqrt{\sigma_l^2 + \sigma_h^2 - \sigma_l \sigma_h} = S_y$$

$$\sigma' = \sqrt{\left(\frac{pr}{t}\right)^2 + \left(\frac{pr}{2t}\right)^2 - \frac{p^2 r^2}{2t^2}} \Rightarrow \frac{\sqrt{3}}{2}\frac{pr}{t} = \sigma'$$

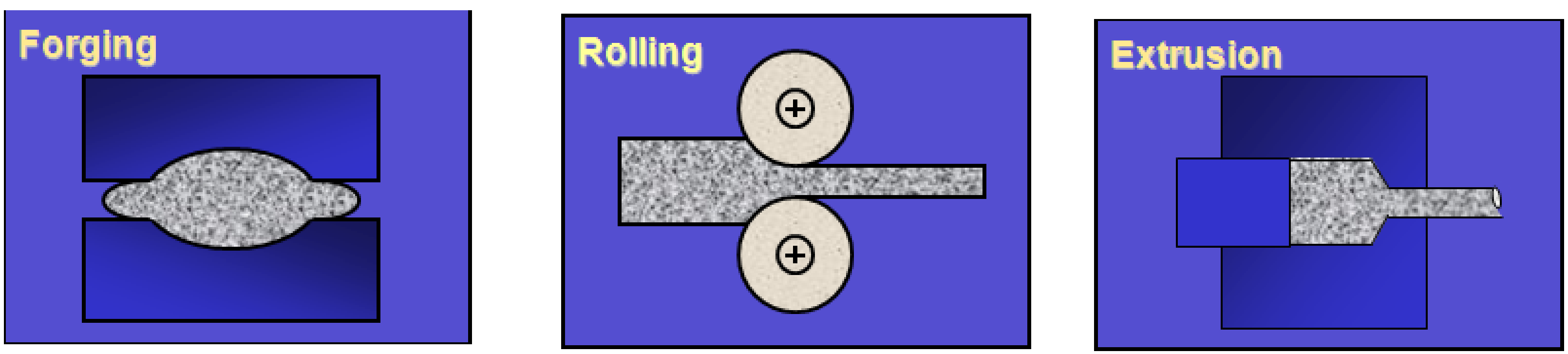
$$\frac{\sqrt{3}}{2} \times \frac{p(10)}{0.1} = 20000 \Rightarrow p = 231\, psi$$

معیار Von Mises واقع بینانه تر عمل می کند.

Flow stress

True stress required to maintain plastic deformation at a given true strain.

محاسبه این تنش برای فرآیندهای شکل دهی پلاستیک فلزات مانند forging, rolling و extrusion مورد نیاز است. لازم به ذکر است که در این فرآیندها ما به دنبال شکست ماده هستیم.



Analysis of bulk deformation

این آنالیز بر اساس روش انرژی انجام می شود.

Work done on the material ≥ Work of deformation

$$U_{total} = U_{ideal} + U_{friction} + U_{redundant}$$

($U_{friction} \to 0$, $U_{redundant} \to 0$)

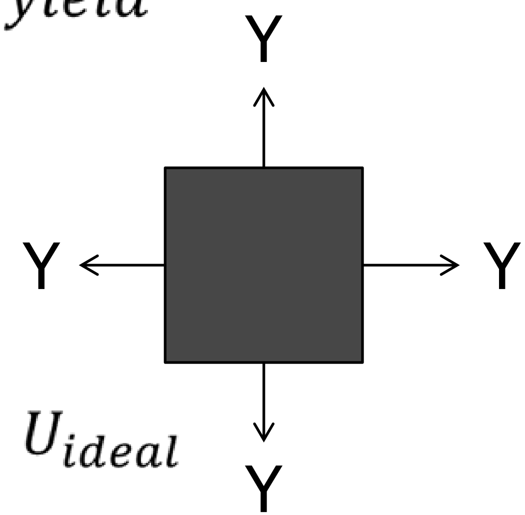
In this course, $U_{friction}$ and $U_{redundant}$ are negligible.

⟶ Work done = Energy dissipated to deform

مثال

انرژی کل را برای تغییر شکل یک جداره ی نازک کروی محاسبه کنید.

$$Thin-walled\ sphere \rightarrow plane\ stress\ at\ yield$$



$$U_{total} = U_{ideal} + \cancelto{0}{U_{friction}} + \cancelto{0}{U_{redundant}} = U_{ideal}$$

$$U_{ideal} = specific\ energy$$

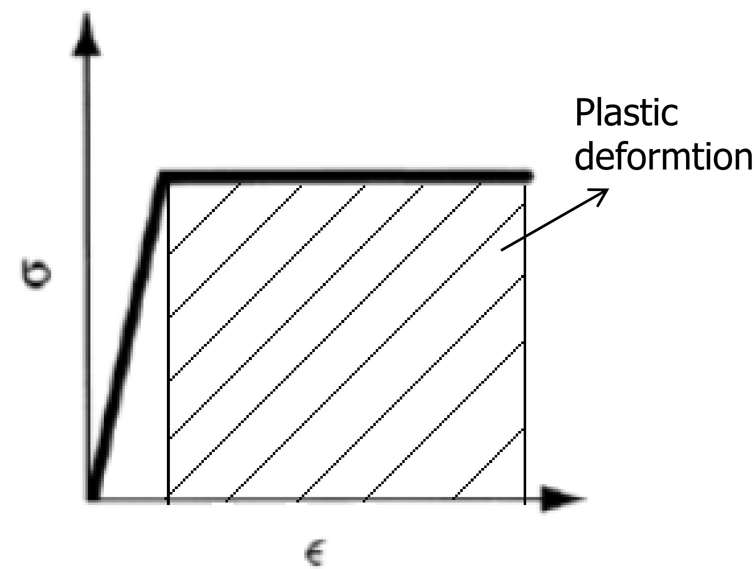
اگر ماده را Elastic-perfectly plastic فرض کنیم داریم:

$$U_x = \int_0^{\varepsilon_x} \sigma_x \, d\varepsilon_x \quad , U_y = \int_0^{\varepsilon_y} \sigma_y \, d\varepsilon_y$$

$$U_{total} = 2\int_0^{\varepsilon_x} \sigma_x \, d\varepsilon_x = 2Y\int_0^{\varepsilon_x} d\varepsilon_x = 2Y\varepsilon_x$$

$$\varepsilon_x = \varepsilon_y = \ln\left(\frac{r_f}{r_i}\right)$$

$$U_{total} = 2Y\ln\left(\frac{r_f}{r_i}\right)$$



$$Total\ Energy\ is\ given\ by{:}\ \ W = U.V$$

$$V = 4\pi r_0^2 t \longrightarrow W = 8\pi Y r_0^2 t \ln\left(\frac{r_f}{r_i}\right)$$

محاسبه انرژی با در نظر گرفتن مدل ماده با رفتار Power Law

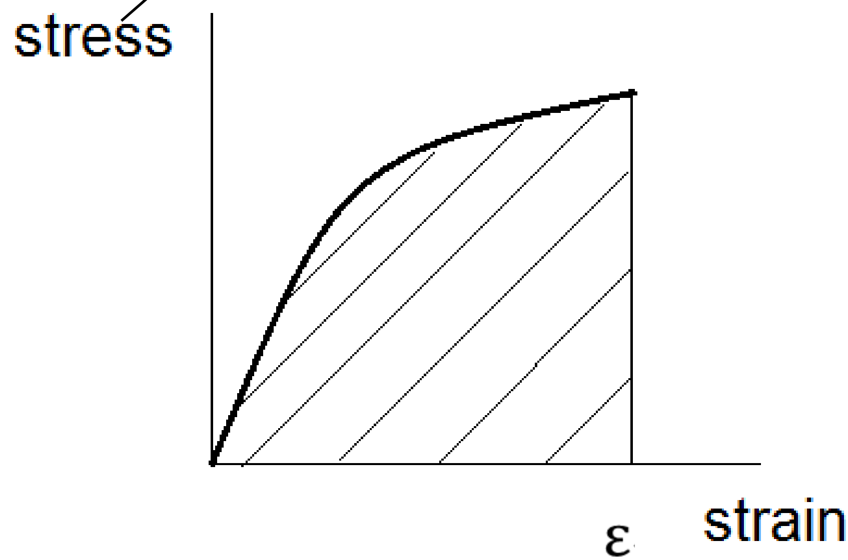
$$U = \int_0^{\varepsilon_1} \sigma \, d\varepsilon$$

$$U = \int_0^{\varepsilon_1} k\varepsilon^n \, d\varepsilon$$

$$U = k\int_0^{\varepsilon_1} \varepsilon^n \, d\varepsilon = \frac{k\varepsilon_1^{n+1}}{n+1}$$

$$\bar{Y} = \frac{k\varepsilon_1^n}{n+1} = \frac{\sigma_1}{n+1} \dashrightarrow U = \bar{Y}\varepsilon$$

$\sigma = k\varepsilon^n$



Strain hardening has the effect that specific energy increases as strain increases.

مثال

مثال قبل را با فرض Power Law حل کنید.

$$specific\ energy = \int_0^{\varepsilon_x} \sigma_x \, d\varepsilon_x + \int_0^{\varepsilon_y} \sigma_y \, d\varepsilon_y = U_{total} = 2\int_0^{\varepsilon_x} \sigma_x \, d\varepsilon_x$$

$$\sigma_x = k\varepsilon_x^n \ , \ \varepsilon_x = \ln\left(\frac{r_f}{r_i}\right)$$

$$U_{total} = 2k\int_0^{\varepsilon_x} \varepsilon_x^n \, d\varepsilon = \frac{2k\varepsilon_x^{n+1}}{n+1} \longrightarrow U_{total} = \frac{2k}{n+1}\ln\left(\frac{r_f}{r_i}\right)^{n+1}$$

$$W = U.V \ , V = 4\pi r_0^2 t$$

$$\longrightarrow W = \frac{8\pi r_0^2 tk}{n+1}\ln\left(\frac{r_f}{r_i}\right)^{n+1}$$